The remaining three equations are of the form

$$\frac{\partial^2 e_{ii}}{\partial x^k \partial x^k} - \frac{\partial^2 e_{ik}}{\partial x^k \partial x^i} - \frac{\partial^2 e_{ki}}{\partial x^i \partial x^k} + \frac{\partial^2 e_{kk}}{\partial x^i \partial x^i} + \left\{ {s \atop kk} \right\}\left(2\frac{\partial e_{si}}{\partial x^i} - \frac{\partial e_{ii}}{\partial x^s}\right) - \left\{ {s \atop ki} \right\}\left(\frac{\partial e_{si}}{\partial x^k} + \frac{\partial e_{sk}}{\partial x^i} - \frac{\partial e_{ik}}{\partial x^s}\right) + \left\{ {s \atop ii} \right\}\left(2\frac{\partial e_{sk}}{\partial x^k} - \frac{\partial e_{kk}}{\partial x^s}\right) - \left\{ {s \atop ik} \right\}\left(\frac{\partial e_{sk}}{\partial x^i} + \frac{\partial e_{si}}{\partial x^k} - \frac{\partial e_{ik}}{\partial x^s}\right) - 2\left(\left\{ {s \atop kk} \right\}\left\{ {t \atop ii} \right\} - \left\{ {s \atop ki} \right\}\left\{ {t \atop ik} \right\}\right) e_{st} = 0 \quad \text{(2b)}$$

In the preceding equations $i, k, s, t = 1, 2, 3$. The repeated scripts imply summation only for s and t, but not for i and k; furthermore $i \neq k$ because they represent two distinct coordinates. The quantities

$$\left\{ {i \atop ks} \right\}$$

are the Christoffel symbols of the second kind.[7] Equations (2) are valid for any curvilinear coordinate systems within the framework ofthe linear theory of strains. Corresponding expressions in somewhat different form can be found in Ref. 8.

Orthogonal Curvilinear Coordinates in Three Dimensions

The form of the six equations are the same as given previously in (2). Since the metric coefficients $g_{ik} = 0$ whenever $i \neq k$, the Christoffel symbols are expressed in terms of the coefficients $h_i = (g_{ii})^{1/2}$.

$$\left.\begin{aligned} &\left\{ {k \atop ij} \right\} = 0 && (i \neq j \neq k) \\ &\left\{ {k \atop ii} \right\} = -\frac{h_i}{h_k{}^2}\frac{\partial h_i}{\partial x^k} && (i \neq k) \\ &\left\{ {i \atop ik} \right\} = \left\{ {i \atop ki} \right\} = \frac{1}{h_i}\frac{\partial h_i}{\partial x^k} && (i \neq k \text{ or } i = k) \end{aligned}\right\} \quad (3)$$

The tensor components of strain e are related to the physical components ϵ by the relations

$$e_{ik} = h_i h_k \epsilon_{ik}$$

Plane Orthogonal Curvilinear Coordinates

For plane coordinates, the system of equations (2) reduces to a single equation

$$\frac{\partial^2 e_{ii}}{\partial y^k \partial y^k} - 2\frac{\partial^2 e_{ik}}{\partial y^i \partial y^k} + \frac{\partial^2 e_{kk}}{\partial y^i \partial y^i} + \frac{1}{h_i}\frac{\partial h_i}{\partial y^i}\left(2\frac{\partial e_{ik}}{\partial y^k} - \frac{\partial e_{kk}}{\partial y^i}\right) + \frac{1}{h_k}\frac{\partial h_k}{\partial y^k}\left(2\frac{\partial e_{ik}}{\partial y^i} - \frac{\partial e_{ii}}{\partial y^k}\right) - \frac{h_i}{h_k{}^2}\frac{\partial h_i}{\partial y^k}\frac{\partial e_{kk}}{\partial y^k} - \frac{h_k}{h_i{}^2}\frac{\partial h_k}{\partial y^i}\frac{\partial e_{ii}}{\partial y^i} - 2\left(\frac{1}{h_i}\frac{\partial h_i}{\partial y^k}\frac{\partial e_{ii}}{\partial y^k} + \frac{1}{h_k}\frac{\partial h_k}{\partial y^i}\frac{\partial e_{kk}}{\partial y^i}\right) + \frac{2}{h_i{}^2}\left[\frac{h_k}{h_i}\frac{\partial h_k}{\partial y^i}\frac{\partial h_i}{\partial y^i} + \left(\frac{\partial h_i}{\partial y^k}\right)^2\right] e_{ii} + \frac{2}{h_i h_k}\left[\frac{\partial h_i}{\partial y^k}\frac{\partial h_k}{\partial y^i} - \frac{\partial h_k}{\partial y^k}\frac{\partial h_i}{\partial y^i}\right] e_{ik} + \frac{2}{h_k{}^2}\left[\frac{h_i}{h_k}\frac{\partial h_k}{\partial y^k}\frac{\partial h_i}{\partial y^k} + \left(\frac{\partial h_k}{\partial y^i}\right)^2\right] e_{kk} = 0 \quad (4)$$

This equation is similar to that of Vlasov's Eq. (15), but Vlasov did not include the terms involving e_{ii}, e_{ik}, and e_{kk}.

Example: Plane Polar Coordinates

In this case, $h_r = 1$ and $h_\theta = r$. Equation (4) takes the form

$$\frac{1}{r^2}\cdot\frac{\partial^2 \epsilon_r}{\partial \theta^2} + \frac{\partial^2 \epsilon_\theta}{\partial r^2} - \frac{2}{r}\cdot\frac{\partial^2 \epsilon_{r\theta}}{\partial r \partial \theta} + \frac{1}{r}\frac{\partial}{\partial r}(2\epsilon_\theta - \epsilon_r) - \frac{2}{r^2}\cdot\frac{\partial \epsilon_{r\theta}}{\partial \theta} = 0 \quad (5)$$

This same example has been considered by Vlasov, but his equation cannot be satisfied by the strain-displacement relations, which is a fundamental requirement for any equation of compatibility

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Erratum: "Ablation in a High Shear Environment"

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IN the caption of Fig. 8 of this article, "$p_s = 15$ atm" should read "$p_s = 47$ atm."

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Errata: "Extension of Second-Order Theory of Entry Mechanics to Oscillatory Entry Solutions"

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EQUATION (5) should read

$$\ln\left[\frac{(V^2/gR_0)}{\dfrac{V_f{}^2}{gR_0}}\right] = \frac{(C_DA/m\beta)(\theta - \theta_f)}{\dfrac{1}{2}\left(\dfrac{L}{D}\right)\left(\dfrac{C_DA}{m\beta}\right) - \left(\dfrac{1}{\beta R_0}\right)\dfrac{\cos\theta}{\rho}\left(\dfrac{gR_0}{V^2} - 1\right)} \quad (5)$$

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